Bubble Growth in Cosmological 1st Order Phase Transitions:

Runaway Walls vs Hydrodynamic Obstructions

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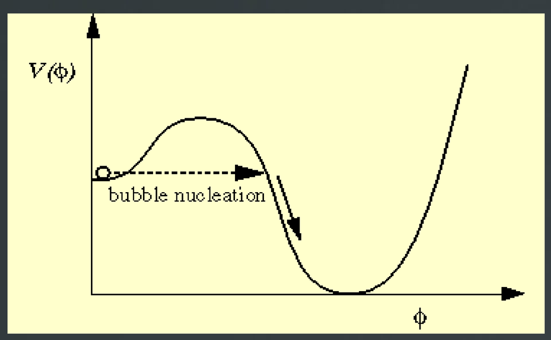
Motivation

Why Bubbles? ➔ **Phase Transitions in the Early Universe**

If 1st Order...



Nucleation & Growth of True Vacuum Bubbles in False Vacuum Sea.

(Nucleation Temperature T_N)

⇨

Bubble Growth in Cosmological Phase Transitions Relevant for:

- ***Electroweak Baryogenesis***

  1st Order Phase Transition = 3rd Sakharov condition.

  Viable Baryogenesis ➔ $v_w < c_s$ ($v_w << 1$ is Favoured)

- ***Stochastic Background of Gravitational Waves***

  Bubble Collisions ($\propto v_w^4$) + Turbulence in Plasma

Formalism

❶ *Matching Equations Across the Bubble Wall.*

System **"Higgs Wall – Plasma"** ⇨

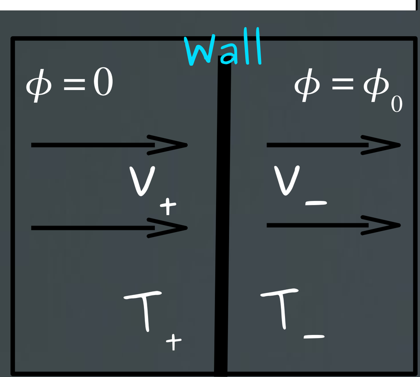
$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\tfrac{1}{2}\partial_\rho\phi\partial^\rho\phi - V_0(\phi)\right)$$
$$T^{Plasma}_{\mu\nu} = w u_\mu u_\nu - g_{\mu\nu} p$$

Energy-Momentum Conserved Across Wall
\+
Steady State (Wall Reference Frame)

$$w_+ v_+^2\gamma_+^2 + p_+ - V_0(0) = w_- v_-^2\gamma_-^2 + p_- - V_0(\phi_0)$$
$$w_+ v_+\gamma_+^2 = w_- v_-\gamma_-^2$$

With $V_0(0) - V_0(\phi_0) \equiv \epsilon$

$$v_+ = \frac{1}{1+\alpha_+}\left(\frac{v_-}{2} + \frac{1}{6v_-} \pm \sqrt{\left(\frac{v_-}{2}+\frac{1}{6v_-}\right)^2 + \alpha_+^2 + \frac{2}{3}\alpha_+ - \frac{1}{3}}\right) \qquad \alpha_+ \equiv \frac{\epsilon}{a_+ T_+^4}$$



Fluid Equations

\+

Boundary Conditions v_+, v_-

❷ *Fluid Equations for Plasma.* $\left(\begin{array}{l}\partial_\mu T^{\mu\nu}_{Plasma} = 0 \\ v(r,t) = v(\xi = r/t)\end{array}\right)$

$$\frac{1-\xi v(\xi)}{1-v^2(\xi)}\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v = 2\frac{v(\xi)}{\xi} \quad + \quad w(\xi) = w_0 \exp\left[\left(1+\frac{1}{c_s^2}\right)\int_{v_0}^{v(\xi)} \gamma^2 \mu \, dv\right]$$

❸ *Fluid Solutions.*

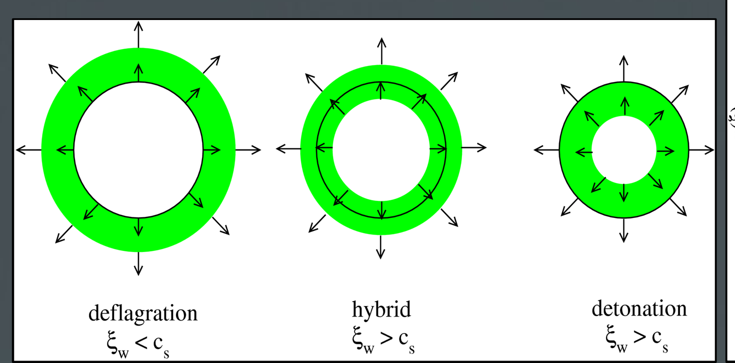
➔ ***Deflagrations:*** $c_s > v_- = v_w > v_+$

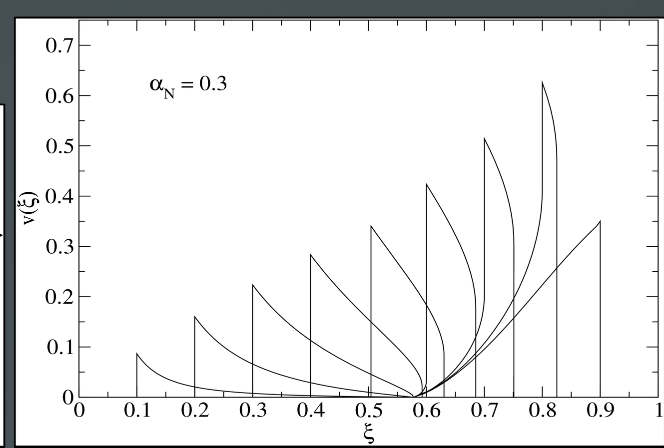
- **Subsonic v_w. Fluid at Rest Behind Bubble Wall.**
- **Compression Wave in Front of Wall.** $T_+ > T_N$

➔ ***Detonations:*** $v_w = v_+ > v_- > c_s$

- **Supersonic v_w. Fluid at Rest in Front of Wall.**
- **Rarefaction Wave Behind Wall.** $T_+ = T_N$

➔ ***Hybrids.***





❹ *Higgs Equation of Motion.* (*Needed to fix ξ_w*)

$$\partial_\mu\partial^\mu\phi + \frac{\partial\mathcal{F}}{\partial\phi} - \mathcal{K}(\phi) = 0$$

Phenomenological Ansatz

$$\mathcal{K}(\phi) = T_N \tilde{\eta}\frac{u^\mu\partial_\mu\phi}{\sqrt{1+(\lambda_\mu u^\mu)^2}}$$

$$\partial_z^2\phi - \frac{\partial\mathcal{F}}{\partial\phi} = T_N\tilde{\eta} v \partial_z\phi \quad\longrightarrow\quad \alpha_+ - \alpha_c = \eta\frac{\alpha_+}{\alpha_N}\langle v\rangle$$

With $\mathcal{K}(\phi) = -\sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3p}{(2\pi)^3 2E_i}\delta f_i(p)$

η *Friction Parameter*

Hydrodynamic Obstruction

Higgs EoM: ***Steady State Solution***

$$\partial_\mu\partial^\mu\phi + \frac{\partial\mathcal{F}}{\partial\phi} - \mathcal{K}(\phi) = 0 \;\longrightarrow\; F_{dr} = \int dz\,\partial_z\phi\frac{\partial\mathcal{F}}{\partial\phi} = \int dz\,\partial_z\phi\,\mathcal{K}(\phi) = F_{fr}$$

Suppose Deflagration Solution:

⇨ **As Wall Moves, Reheats Plasma in Front** ($T_+ > T_N$).

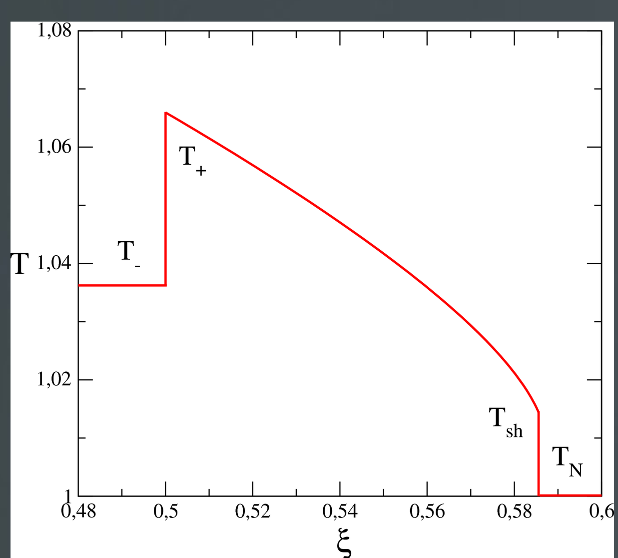
⇨ **As v_w Increases, T_+ Raises** ($T_+ \uparrow$ if $v_w \uparrow$).

⇨ **As T_+ Raises, F_{dr} Decreases** ($F_{dr} \downarrow$ if $T_+ \uparrow$).

⇩

It could happen that $F_{dr} \to 0$ for $v_w < c_s$

Hydrodynamic Obstruction!!



Criterium for Hydrodynamic Obstruction:

$$\alpha_N < \alpha_c \,\mathrm{Exp}\left(\frac{16}{3(1+\alpha_c)}\Omega(\alpha_c)\right) \quad \text{With} \quad \Omega(\alpha_c) \equiv \sqrt{\frac{\alpha_c}{2} - \frac{3}{10}\alpha_c - \frac{1}{5}\alpha_c^{3/2}}$$

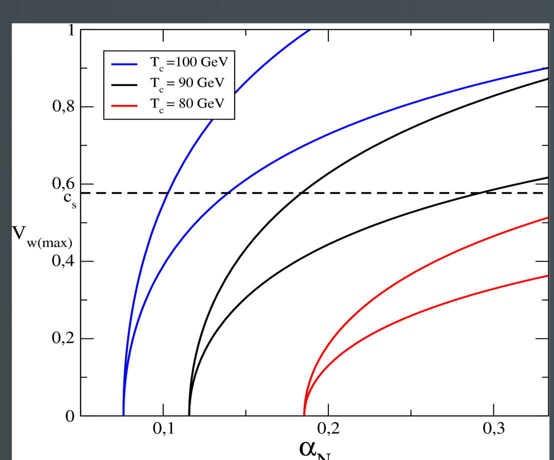
$v_w = c_s$ *Saturates Inequality*

If Obstruction Occurs, $T_+ > T_c > T_-$

- ***Lower Limit*** ($T_+ = T_c$): $v_w^{max} = \sqrt{\frac{1}{6\alpha_c}\mathrm{Log}\left(\frac{T_c}{T_N}\right)}$

  ($v_w << 1$ approximation)

- ***Upper Limit*** ($T_- = T_c$): $v_w^{max} = \sqrt{\frac{1}{3\alpha_c}\mathrm{Log}\left(\frac{T_c}{T_N}\right)}$



Higgs EoM Revisited:

$$\dot{v_w} \propto (\mathcal{F}_- - \mathcal{F}_+)(T) - v_w\eta \longrightarrow \dot{v_w} \propto (\mathcal{F}_- - \mathcal{F}_+)(T_N) - v_w^2\kappa - v_w\eta$$

Runaway Walls

Higgs EoM:

$$\partial_\mu\partial^\mu\phi + \frac{\partial\mathcal{F}}{\partial\phi} - \mathcal{K}(\phi) = 0 \qquad F_{fr} = \int dz\,\partial_z\phi\,\mathcal{K}(\phi) \qquad F_{dr} = \int dz\,\partial_z\phi\frac{\partial\mathcal{F}}{\partial\phi}$$

Driving Force — Friction Force

⇨ ***For $v_w << 1$ (Low Velocity Limit)*** ⟶ $F_{fr} \propto \gamma_w v_w \sim v_w$

⇨ ***For $v_w \to 1$ (Ultrarelativistic Limit)*** ⟶ $F_{fr} \to F_{fr}^{max} = Cte$

D. Bodecker and G. D. Moore, JCAP **0905** (2009) 009

- ***If $F_{dr} < F_{fr}^{max}$:***

  Friction Equilibrates with Driving Force.

  Steady State Solution.

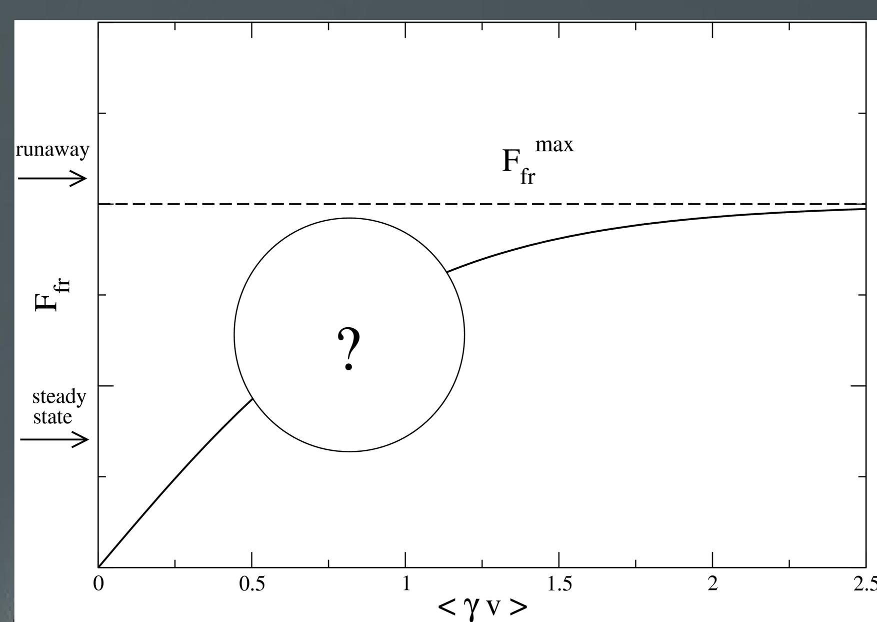
  (Deflagrations/Hybrids/Detonations)

- ***If $F_{dr} > F_{fr}^{max}$:***

  Friction does not Equilibrate Driving Force.

  Runaway Solution.

  Continuous Acceleration (No Steady State)



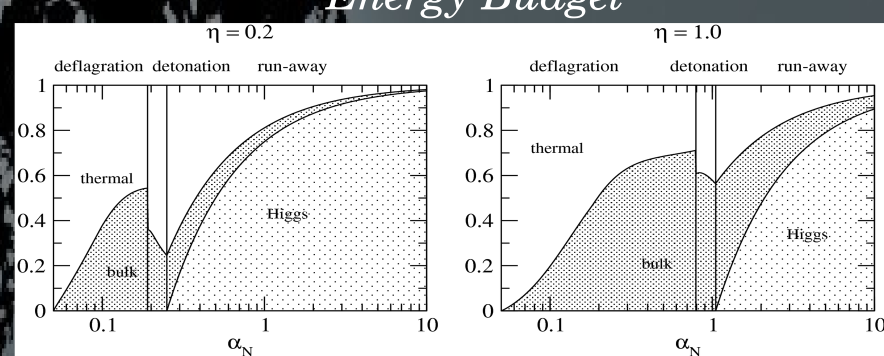
Runaway Criterium ($F_{dr} > F_{fr}^{max}$)

$$\alpha_N > \alpha_\infty \equiv \frac{30}{\pi^2}\left(\frac{\phi_0}{T_N}\right)^2\frac{\sum_{light\to heavy} c_i\|N_i\| y_i^2}{\sum_{light} c_i'\|N_i\|}$$

If $\alpha_N > \alpha_\infty$, Some Energy into Higgs Field

Energy Budget Altered!!

Energy Budget



Conclusions

Even for Vanishing Friction, Small v_w Still Possible (depends on α_N).
(*Hydrodynamic Obstruction to Larger v_w*)

⟹ Favours Baryogenesis

Runaway Solutions Change Energy Budget of 1st Order Phase Transitions.
(Also, Detonation Parameter Space Drastically Reduced).

⟹ May Change Gravitational Wave Spectrum (Mainly Turbulence)